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LETTER TO THE EDITOR

Counting of ghosts in a quantised antisymmetric tensor gauge field of third rank

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Abstract. By introducing the BRS transformation and Lagrangian multiplier fields, the covariant canonical quantisation is carried out for an antisymmetric tensor gauge field of third rank. The non-propagating character of the field is assured from the viewpoint of the 'quartet mechanism' of Kugo and Ojima. The correct number of ghosts is counted from the effective Lagrangian.

In a recent Letter in this journal, Namazie and Storey (1980) have discussed the secondary and further Faddeev–Popov (FP) ghosts occurring in covariantly quantised gauge theories. They have shown that two second-rank tensor FP ghosts, four FP vector ghosts and eight scalar ghosts are required in order to quantise an antisymmetric tensor gauge field of third rank $A_{\mu\nu\lambda}$. The counting of total degrees of freedom is $4-(2\times 6)+(4\times 4)-8=0$ and seems to be in accordance with the fact that $A_{\mu\nu\lambda}$ is non-propagating. Ward identities need not be invoked to obtain the correct count, contrary to the case of the second-rank antisymmetric tensor gauge field where one redundant degree must decouple due to Ward identity in order to assure the unitarity of the S matrix as shown by Townsend (1979). In this Letter, we shall show that the counting of ghosts number by Namazie and Storey (1980) is incorrect.

In a recent paper, the author (Kimura 1980) has carried out the covariant canonical quantisation of the antisymmetric tensor gauge field of second rank with the help of the 'quartet mechanism' of Kugo and Ojima (1978, 1979) and has shown that all unphysical components are confined. Accordingly, the difficulty of Townsend (1979) is removed and the correct number of FP ghosts is obtained using this formalism. Here we shall extend the formalism to the case of the antisymmetric tensor gauge field of third rank.

We start from the Lagrangian density

$$\begin{aligned} L_0 &= -\frac{1}{2\times 4!}F_{\mu\nu\lambda\rho}F^{\mu\nu\lambda\rho} \\ &= -\tfrac{1}{12}\partial_\mu A_{\nu\lambda\rho}\partial^\mu A^{\nu\lambda\rho}+\tfrac{1}{4}\partial_\mu A_{\nu\lambda\rho}\partial^\nu A^{\mu\lambda\rho} \end{aligned} \tag{1}$$

where $F_{\mu\nu\lambda\rho}=\partial_\mu A_{\nu\lambda\rho}-\partial_\nu A_{\lambda\rho\mu}+\partial_\lambda A_{\rho\mu\nu}-\partial_\rho A_{\mu\nu\lambda}$ and we use the metric $\eta_{\mu\nu}=(-1,1,1,1)$. The above Lagrangian density is invariant under the gauge transformation

$$A_{\mu\nu\lambda}\to A_{\mu\nu\lambda}+(\partial_\mu\xi_{\nu\lambda}+\partial_\nu\xi_{\lambda\mu}+\partial_\lambda\xi_{\mu\nu}). \tag{2}$$

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To quantise the field we introduce the gauge-fixing terms and corresponding ghost terms and construct the Lagrangian so that it is invariant under the BRS transformation mentioned below. Our required Lagrangian density is

$$
\begin{aligned}
L = L_0 &- \tfrac{1}{2}A^{\mu\nu\lambda}\partial_\mu A_{\nu\lambda} + \tfrac{1}{4}\alpha A_{\mu\nu}A^{\mu\nu} - A^{\mu\nu}\partial_\mu A_\nu - A^\mu \partial_\mu A + \tfrac{1}{2}\alpha' AA \\
&- \tfrac{1}{2}\mathrm{i}(\partial^\mu b_*^{\nu\lambda}\partial_\mu b_{\nu\lambda} - 2\partial^\mu b_*^{\nu\lambda}\partial_\nu b_{\mu\lambda}) - \mathrm{i}b_*^{\mu\nu}\partial_\mu b_\nu + \mathrm{i}b_{\mu\nu}\partial^\mu b_*^\nu \\
&+ \mathrm{i}\beta b_*^\nu b_\nu - \mathrm{i}b_*^\nu \partial_\nu b + \mathrm{i}b^\nu \partial_\nu b_* + (\partial^\lambda c_*^\mu \partial_\lambda c_\mu - \partial^\lambda c_*^\rho \partial_\rho c_\lambda) \\
&+ c_*^\mu \partial_\mu c + c_\mu \partial^\mu c_* - \gamma c_* c + \mathrm{i}\partial^\lambda d_* \partial_\lambda d
\end{aligned}
\tag{3}
$$

where α, α', β and γ are real parameters. We postulate that all field variables are Hermitian. The fields $A_{\mu\nu\lambda}$, $A_{\mu\nu}$, A_ν, A, $c_{*\mu}$, c_ν, c_* and c are subject to Bose statistics, while $b_{*\mu\nu}$, $b_{\lambda\rho}$, $b_{*\mu}$, b_ν, b_*, b, d_* and d are subject to Fermi statistics. The Lagrangian density (3) is invariant under the BRS transformations

$$
\begin{aligned}
&\delta A_{\mu\nu\lambda} = \lambda(\partial_\mu b_{\nu\lambda} + \partial_\nu b_{\lambda\mu} + \partial_\lambda b_{\mu\nu}) \qquad \delta A_\mu = \lambda b_\mu \qquad \delta A_{\mu\nu} = \delta A = 0 \\
&\delta b_{*\mu\nu} = \mathrm{i}\lambda A_{\mu\nu} \qquad \delta b_{\mu\nu} = \mathrm{i}\lambda(\partial_\mu c_\nu - \partial_\nu c_\mu) \qquad \delta b_{*\nu} = \delta b_\nu = 0 \\
&\delta b_* = \mathrm{i}\lambda A, \qquad \delta b = \mathrm{i}\lambda c \\
&\delta c_{*\mu} = \lambda b_{*\mu}, \qquad \delta c_\nu = \lambda\partial_\nu d, \qquad \delta c_* = \delta c = 0 \\
&\delta d_* = \mathrm{i}\lambda c_*, \qquad \delta d = 0
\end{aligned}
\tag{4}
$$

in which the x-independent parameter λ anticommutes with all the Fermi fields and commutes with all the Bose fields. The introduction of $A_{\mu\nu}$, etc, assures the off-shell nilpotency of the BRS transformations $\delta'(\delta b_{*\mu\nu}) = 0$, etc.

In order to simplify the discussion we take

$$
\beta = \gamma = 1, \qquad \alpha' = \alpha,
\tag{5}
$$

by which all fields become simple pole fields except for $A_{\mu\nu\lambda}$. The equations of motion derived from (3) with (5) are

$$\partial\rho A_{\rho\mu\nu} + \alpha A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{6}$$

$$\partial^\lambda A_{\lambda\mu} = \partial_\mu A, \qquad \partial^\lambda A_\lambda + \alpha A = 0 \tag{7}$$

$$a^\lambda b_{\lambda\mu} + b_\mu - \partial_\mu b = 0, \qquad \partial^\lambda b_{*\lambda\mu} + b_{*\mu} - \partial_\mu b_* = 0 \tag{8}$$

$$\partial^\lambda b_\lambda = 0, \qquad \partial_\lambda b_*^\lambda = 0 \tag{9}$$

$$\partial_\lambda c^\lambda + c = 0, \qquad \partial_\lambda c_*^\lambda + c_* = 0 \tag{10}$$

$$\Box A_{\mu\nu\lambda} - (1-\alpha)(\partial_\mu A_{\nu\lambda} + \partial_\nu A_{\lambda\mu} + \partial_\lambda A_{\mu\nu}) = 0 \tag{11}$$

$$\Box \Phi_{I'} = 0 \tag{12}$$

where $\Phi_{I'}$ stands for field variables except $A_{\mu\nu\lambda}$. The field $A_{\mu\nu\lambda}$ is a dopole field except for the case of $\alpha = 1$ (Feynman gauge).

There are nine pairs of second-class constants between the field variables and their conjugate momenta. To get a consistent theory we employ the new Dirac bracket method following the procedure proposed by us (Endo and Kimura 1979). Adopting the equal-time (anti-) commutators obtained by this procedure, we have the following four-dimensional (anti) commutation relations among the field variables:

$$
\begin{aligned}
[A_{\mu\nu\lambda}(x), A_{\rho\sigma\omega}(y)] &= \mathrm{i}K_{\mu\nu\lambda,\rho\sigma\omega}D(x-y) \\
&\quad + \mathrm{i}(1-\alpha)\sum_{\rho\sigma\omega}^{\text{cyclic}}[(K_{\mu\nu,\rho\sigma}\partial_\lambda + K_{\nu\lambda,\rho\sigma}\partial_\mu + K_{\lambda\mu,\rho\sigma}\partial_\nu)\partial_\omega]\tilde{D}(x-y) \\
[A_{\mu\nu\lambda}(x), A_{\rho\sigma}(y)] &= \mathrm{i}(K_{\mu\nu,\rho\sigma}\partial_\lambda + K_{\nu\lambda,\rho\sigma}\partial_\mu + K_{\lambda\mu,\rho\sigma}\partial_\nu)D(x-y) \\
[A_\mu(x), A_{\rho\sigma}(y)] &= \mathrm{i}\{b_\mu(x), b_{*\rho\sigma}(y)\} = -\mathrm{i}\{b_{*\mu}(x), b_{\rho\sigma}(y)\} \\
&= \mathrm{i}(\eta_{\mu\rho}\partial_\sigma - \eta_{\mu\sigma}\partial_\rho)D(x-y) \\
[A_\mu(x), A_\rho(y)] &= \mathrm{i}\alpha\eta_{\mu\rho}D(x-y) \qquad (13) \\
[A(x), A_\mu(y)] &= -[c(x), c_{*\mu}(y)] = -[c_*(x), c_\mu(y)] = \mathrm{i}\{b(x), b_{*\mu}(y)\} \\
&= -\mathrm{i}\{b_*(x), b_\mu(y)\} = -\mathrm{i}\partial_\mu D(x-y) \\
\{b_{\mu\nu}(x), b_{*\rho\sigma}(y)\} &= K_{\mu\nu,\rho\sigma}D(x-y) \\
\{b(x), b_*(y)\} &= -\{d(x), d_*(y)\} = D(x-y) \\
[c_\mu(x), c_{*\rho}(y)] &= -\mathrm{i}\eta_{\mu\rho}D(x-y);
\end{aligned}
$$

all the other (anti-) commutators vanish. In the above, $K_{\mu\nu,\lambda\rho} = \eta_{\mu\lambda}\eta_{\nu\rho} - \eta_{\mu\rho}\eta_{\nu\lambda}$ and $K_{\mu\nu\lambda,\rho\sigma\omega} = \Sigma_{\rho\sigma\omega}^{\text{perm}}(\pm)\eta_{\mu\rho}n_{\nu\sigma}\eta_{\lambda\omega}$ where $+(-)$ is taken according to even (odd) permutations of ρ, σ, ω. $D(x)$ and $\tilde{D}(x)$ are defined by

$$D(x) = -\mathrm{i}(2\pi)^{-3}\int \mathrm{d}^4k\epsilon(k_0)\delta(k^2)\,\mathrm{e}^{\mathrm{i}kx}$$

$$\tilde{D}(x) = -\mathrm{i}(2\pi)^{-3}\int \mathrm{d}^4k\epsilon(k_0)\delta'(k^2)\,\mathrm{e}^{\mathrm{i}kx}$$

and $\tilde{D}(x)$ satisfies $\Box\tilde{D}(x) = D(x)$.

Following the conventional procedure we get the Noether current of the BRS transformation

$$
\begin{aligned}
J_\mu = {}& \tfrac{1}{2}\partial^\lambda b^{\rho\sigma}\partial_\mu A_{\lambda\rho\sigma} - \tfrac{1}{2}(\partial^\lambda b^{\rho\sigma} + 2\partial^\rho b^{\sigma\lambda})\partial_\lambda A_{\mu\rho\sigma} - \tfrac{1}{2}A^{\lambda\rho}\partial_\mu b_{\lambda\rho} + A^{\lambda\rho}\partial_\lambda b_{\mu\rho} + b^\nu A_{\mu\nu} - Ab_\mu \\
& + \tfrac{1}{2}c^{\lambda\rho}\partial_\mu b_{*\lambda\rho} - c^{\lambda\rho}\partial_\lambda b_{*\mu\rho} - b_*^\nu c_{\mu\nu} - \partial^\nu dc_{*\mu\nu} + cb_{*\mu} + c_*\partial_\mu d
\end{aligned} \qquad (14)
$$

which satisfies the conservation equation $\partial^\mu J_\mu = 0$. With the help of the equations of motion, the corresponding conserved charge is given by

$$Q_{\mathrm{B}} = \int \mathrm{d}^3x J_0(x) = \int \mathrm{d}^3x(\tfrac{1}{2}b_{\mu\nu}\partial_0 A^{\mu\nu} + b\partial_0 A + c_\mu\partial_0 b_*^\mu + c_*\partial_0 d). \qquad (15)$$

TheQ_{B} generates the BRS transformation given by (4):

$$\delta\Phi_I(x) = [\mathrm{i}\lambda Q_{\mathrm{B}}, \Phi_I(x)]. \qquad (16)$$

We shall now consider the mode counting in the four-dimensional momentum space by referring to a Lorentz frame such that $p_1 = p_2 = 0$ and $p_3 > 0$. By taking account of the equations of motion (6)–(10), in our Lorentz frame, all field variables can be expressed in terms of the following 32 independent variables:

$$\chi_k\colon -\mathrm{i}A_{012}/p_0,\ -\mathrm{i}A_{023}/p_0,\ -\mathrm{i}A_{031}/p_0,\ A_0,\ c_{*1},\ c_{*2},\ c_{*3},\ -\mathrm{i}c_0/p_0$$
$$\beta_k\colon A_{12},\ A_{23},\ A_{31},\ -\mathrm{i}A_{03}/p_0,\ -c_1,\ -c_2,\ \mathrm{i}c/p_0,\ -c_*$$
$$\gamma_k\colon b_{12},\ -\mathrm{i}b_2/p_0,\ \mathrm{i}b_1/p_0,\ b_0,\ b_{*1},\ b_{*2},\ b_{*3},\ d$$
$$\gamma_{*k}\colon b_{*12},\ b_{*23},\ b_{*31},\ -\mathrm{i}b_{*03}/p_0,\ \mathrm{i}b_{01}/p_0,\ \mathrm{i}b_{02}/p_0,\ \mathrm{i}b_{03}/p_0,\ -d_*.$$

From the four-dimensional Fourier transforms of (13), we have the following (anti-) commutators between the independent field variables:

$$\begin{array}{c|cc|cc} & \chi_l^\dagger(q) & \beta_l^\dagger(q) & \gamma_l^\dagger(q) & \gamma_{*l}^\dagger(q) \\ \chi_k(p) & \omega_{kl} & \delta_{kl} & & 0 \\ \beta_k(p) & \delta_{kl} & 0 & & \\ \hline \gamma_k(p) & & 0 & 0 & -\mathrm{i}\delta_{kl} \\ \gamma_{*k}(p) & & & \mathrm{i}\delta_{kl} & 0 \end{array} \tag{17}$$

in which the common factor $\theta(p_0)\delta(p^2)\delta^4(p-q)$ is omitted and ω_{kl} need not be specified for the present purpose. In the above, the creation and annihilation operators $\Phi_I^\dagger(p)$, $\Phi_I(p)$ are defined by

$$\Phi_I(x)=(2\pi)^{-3/2}\int \mathrm{d}^4p(\Phi_I(p)\,\mathrm{e}^{\mathrm{i}px}+\Phi_I^\dagger(p)\,\mathrm{e}^{-\mathrm{i}px}).$$

The BRS transformation properties are

$$[\mathrm{i}Q_\mathrm{B},\chi_l]=\gamma_l,\qquad \{\mathrm{i}Q_\mathrm{B},\gamma_{*l}\}=\mathrm{i}\beta_l,\qquad [Q_\mathrm{B},\beta_l]=\{Q_\mathrm{B},\gamma_l\}=0. \tag{18}$$

The above forms of (17) and (18) are the same as in the theory of Kugo and Ojima (1979) for a Yang–Mills field where the 'quartet mechanism' works and where the unphysical components are confined in the physically invisible world by imposing the condition

$$Q_\mathrm{B}|\mathrm{phys}\rangle=0. \tag{19}$$

When interactions are added, either with other fields or in a non-Abelian generalisation, the above formalism is regarded as that of the asymptotic fields. The unitarity of the physical S matrix can be proved in the same way as in the Kugo–Ojima formalism.

Taking $\alpha=1$ and omitting divergence terms, we can rewrite the Lagrangian density (3) as

$$\begin{aligned} L=&-\tfrac{1}{12}\partial^\lambda A^{\mu\nu\rho}\partial_\lambda A_{\mu\nu\rho}-\tfrac{1}{2}\mathrm{i}\partial^\lambda b_*^{\mu\nu}\partial_\lambda b_{\mu\nu}-\tfrac{1}{2}\partial^\lambda A^\mu\partial_\lambda A_\mu-\mathrm{i}\partial^\lambda b_*\partial_\lambda b+\partial^\lambda c_*^\mu\partial_\lambda c_\mu+\mathrm{i}\partial^\lambda d_*\partial_\lambda d\\ &+[\tfrac{1}{4}(\partial^\lambda A_{\lambda\mu\nu}+A_{\mu\nu}-\partial_\mu A_\nu+\partial_\nu A_\mu)(\partial_\rho A^{\rho\mu\nu}+A^{\mu\nu}-\partial^\mu A^\nu+\partial^\nu A^\mu)\\ &+\mathrm{i}(\partial_\lambda b_*^{\lambda\mu}+b_*^\mu-\partial^\mu\eta_*)(\partial^\rho b_{\rho\mu}+b_\mu-\partial_\mu\eta)\\ &+\tfrac{1}{2}(\partial^\lambda A_\lambda+A)(\partial^\rho A_\rho+A)-(\partial_\lambda c_*^\lambda+c_*)(\partial_\rho c^\rho+c)]. \end{aligned} \tag{20}$$

The terms in the square brackets can be ignored by taking account of (6), (8) and (10). Thus we effectively have two tensor ghosts ($b_*^{\mu\nu}$, $b^{\lambda\rho}$), three vector ghosts (A^μ, c_*^μ, c^λ) and four scalar ghosts (b_*, b, d_*, d). It should be noted that our number of ghosts is different from that of Namazie and Storey (1980) (four vector ghosts and eight scalar ghosts). In our counting the Lagrangian multiplier fields have played an important role.

To reduce unphysical components we have adopted the subsidiary condition (19) instead of one of Gupta-Bleuler type, $G^{(+)}|\mathrm{phys}\rangle = 0$ where $G^{(+)}$ means the positive frequency part. Therefore our formalism is promising when we deal with antisymmetric tensor gauge fields in the presence of the gravitational field, since the positive frequency part is, in general, not well defined in this case. Furthermore, the correct counting of ghosts is indispensable in practical calculations (such as the one-loop calculations for the gauge field) in the gravitational background, because the ghosts do not decouple and contribute to loop diagrams, etc.

References

Endo R and Kimura T 1979 *Prog. Theor. Phys.* **61** 1211
Kimura T 1980 *Prog. Theor. Phys.* **64**
Kugo T and Ojima I 1978 *Phys. Lett.* **73B** 1343
—— 1979 *Prog. Theor. Phys. Suppl.* **66**
Namazie M A and Storey D 1980 *J. Phys. A: Math. Gen.* **13** L161
Townsend P K 1979 *Phys. Lett.* **88B** 97